Dynamics and Configuration Control of Two-Body Satellite Systems

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Theme

AN analytical method for devising computer algorithms for the onboard automatic control of relative motion of two-body satellite systems in circular orbit is presented. The method is based on the use of modified Hamiltonians of the relative motion as Liapunov functions for the desired terminal state of the controlled dynamical system. Implicit expressions for the required control forces and corresponding motion trajectories are obtained directly from this formulation. Explicit forms of the corresponding dimensionless control force functions and phase-space trajectories of relative motion can be obtained by numerical integration. Results are presented for the special case of deployment of two-body systems in planar motion from an arbitrary initial condition at satellite separation to terminal system alignment with the local vertical.

Contents

Two-body satellite systems have been studied by several authors as a means for obtaining high-resolution, low-frequency, radio astronomy observations of solar and celestial sources.[1-3] With the two satellites of such a system operating in an interferometric mode, the combination of satellite relative motion together with ground data processing could be used to synthesize a large aperture radio telescope. In most of these studies it was assumed that control of satellite relative motion is to be achieved by means of a connecting tether. Detailed studies of the relative motion of two cable-connected satellites whose mass center moves in a Keplerian orbit have been made.[4,5]

The subject paper of this Synoptic deals with a method for automatic configuration control of two-body satellite systems whose mass center moves in a circular orbit. The method is based on the Hamiltonian formulation of the satellite relative motion problem. It is shown in the paper that specialization to the case of planar system motion ($\phi = 0$, see Fig. 1) yields the equations

$$\begin{aligned} \xi'' + [1 - \{(1+\theta')^2 + 3\cos^2\theta\}]\xi &= F_\xi/m^*\omega^2 l \\ (\theta'' + \tfrac{3}{2}\sin 2\theta)\xi + 2(1+\theta')\xi' &= F_\theta/m^*\omega^2 l \end{aligned} \tag{1}$$

where primes indicate differentiation with respect to $\tau = \omega t$

$$m^* = m_1 m_2/(m_1 + m_2) \tag{2}$$

represents the so-called "reduced mass" of the two-body system, $\omega = (GM/r^3)^{1/2}$ its orbital angular velocity, and l a suitable reference length (in the paper, the baseline length of the deployed system). These equations can be written in the Euler-Lagrange form as

$$\begin{aligned} (\partial L/\partial \xi')' - \partial L/\partial \xi &= Q_\xi = F_\xi l \\ (\partial L/\partial \theta')' - \partial L/\partial \theta &= Q_\theta = F_\theta \xi l \end{aligned} \tag{3}$$

where L represents the Lagrangian

$$L = (m^*/2)\omega^2 l^2[\xi'^2 + \xi^2\{(1+\theta')^2 + (1+3\cos 2\theta)/2\}] \tag{4}$$

and Q_ξ, Q_θ generalized torques. The corresponding Hamiltonian, expressed in terms of the state variables $\xi, \theta, \xi', \theta'$, is

$$H = (m^*/2)\omega^2 l^2\{\xi'^2 + \xi^2(\theta'^2 - 3\cos^2\theta)\} \tag{5}$$

Now, for the purpose of deriving the fundamental control algorithm, consider the following argument. Assume that the control torques Q_ξ and Q_θ in Eq. (3) contain a component which is derivable from a "potential function" $C(\xi, \theta)$ as follows

$$Q = -\partial C/\partial \xi + \bar{Q}_\xi, \quad Q_\theta = -\partial C/\partial \theta + \bar{Q}_\theta \tag{6}$$

with $\bar{Q}_\xi$ and $\bar{Q}_\theta$ representing the components not derivable from a potential. The equations of motion (1) can then be written in terms of the modified Lagrangian function

$$\bar{L} = L - C \tag{7}$$

as

$$\begin{aligned} (\partial \bar{L}/\partial \xi')' - \partial \bar{L}/\partial \xi &= \bar{Q}_\xi = \bar{F}_\xi l \\ (\partial \bar{L}/\partial \theta')' - \partial \bar{L}/\partial \theta &= \bar{Q}_\theta = \bar{F}_\theta \xi l \end{aligned} \tag{8}$$

The Hamiltonian function corresponding to $\bar{L}$ is

$$\bar{H} = H + C \tag{9}$$

as may be readily verified. This function does not depend explicitly on τ. It is known[6] that its rate of change with respect to τ is given by the expression

$$\bar{H}' = \bar{Q}_\xi \xi' + \bar{Q}_\theta \theta'$$

i.e.,

$$\bar{H}' = (\bar{F}_\xi \xi' + \bar{F}_\theta \xi \theta')l \tag{10}$$

Now, if the function $C(\xi, \theta)$ and the control force components $\bar{F}_\xi, \bar{F}_\theta$ can be chosen in such a way that the Hamiltonian

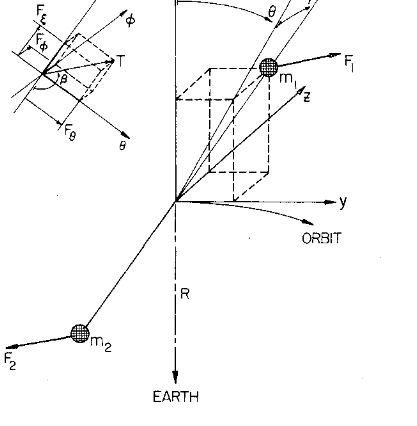

Fig. 1 System configuration and control force ($F_1 = -F_2 = T$).

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function Eqs. (5) and (9) have the property $\bar{H} \geq 0$, and its derivative, Eq. (10), the property $\bar{H}' \leq 0$, with the equality sign in both inequalities holding only for a single state $(\xi^*, \theta^*, \xi'^*, \theta'^*)$ of the dynamical system, then the corresponding control force F, defined by the relations

$$F_\xi = -(1/l)\partial C/\partial\xi + \bar{F}_\xi, \quad F_\theta = -(1/\xi l)\partial C/\partial\theta + \bar{F}_\theta \tag{11}$$

may be expected to drive the system to this particular state. The Hamiltonian $\bar{H}$ can then be interpreted as a Liapunov function for the terminal state of the controlled dynamical system.

For each desired terminal state $(\xi^*, \theta^*, \xi'^*, \theta'^*)$ there are an infinite number of ways in which the conditions $\bar{H} \geq 0$, $\bar{H}' \leq 0$ can be met. This corresponds to the well-known fact that Liapunov functions are not unique. For the deployment problem considered in the paper, with the desired terminal system state defined by the vector

$$(\xi^*, \theta^*, \xi'^*, \theta'^*) = (1, 0, 0, 0) \tag{12}$$

two cases were studied. Here, we consider only one of these (Case A) characterized by the relations

$$\begin{aligned} C &= (m^*/2)\omega^2 l^2\{(\xi-1)^2 + \xi^2(\theta^2 + 3\cos^2\theta)\} \\ \bar{F}_\xi &= -m^*\omega^2 l\nu\xi' \quad \bar{F}_\theta = -m^*\omega^2 l\nu\xi\theta' \end{aligned} \tag{13}$$

with

$$\begin{aligned} \bar{H} &= (m^*/2)\omega^2 l^2\{(\xi-1)^2 + \xi'^2 + \xi^2(\theta^2 + \theta'^2)\} \\ \bar{H}' &= -m^*\omega^2 l^2\nu(\xi'^2 + \xi^2\theta'^2) \end{aligned} \tag{14}$$

and

$$\begin{aligned} F_\xi &= -m^*\omega^2 l\{\xi(1 + \theta^2 + 3\cos^2\theta) - 1 + \nu\xi'\} \\ F_\theta &= -m^*\omega^2 l\xi(\theta - \tfrac{3}{2}\sin 2\theta + \nu\theta') \end{aligned} \tag{15}$$

The constant ν in these expressions represents a proportional control factor the value of which may be chosen arbitrarily. Note that the single state $(\xi^*, \theta^*, \xi'^*, \theta'^*)$ for which both $\bar{H}$ and $\bar{H}'$ vanish is the desired terminal state (1,0,0,0). Substitution of the control force components F_ξ, F_θ from Eq. (15) in the equations of motion (1) yields the reduced dynamical equations

$$\begin{aligned} \xi'' + \nu\xi' + \{2 + \theta^2 - (1+\theta')^2\}\xi &= 1 \\ (\theta'' + \nu\theta' + \theta)\xi + 2(1+\theta')\xi' &= 0 \end{aligned} \tag{16}$$

which govern system motion during deployment. These equations confirm the existence of a single equilibrium configuration ($\xi = 1$, $\theta = 0$) for the controlled dynamical system. For given initial conditions†

$$(\xi_i, \theta_i, \xi_i', \theta_i') = (0, \alpha, \kappa, \lambda) \tag{17}$$

with α, κ, λ arbitrary, and given control force components (15), the equations of motion can be integrated numerically. Some typical results are given in Figs. 2 and 3. They give, respectively, system response $\xi(\tau)$, $\theta(\tau)$ and control force vector quantities $T(\tau)$, $\beta(\tau)$—magnitude and orientation angle with respect to satellite line of centers (see Fig. 1)—for the cases $\nu = 2$, $\kappa = 1$, $\lambda = 0$, and $\alpha = 0°, 30°, 60°$. These results confirm the fact that regardless of the value of the initial angle α, the range function $\xi(\tau)$ increases from zero to one, reaching the terminal value $\xi = 1$ asymptotically for large values of τ. For all practical purposes, however, it may be said that $\xi = 1$ is reached in about one orbit. Figures 2 and 3 also show that the system angular orientation functions $\theta(\tau)$ all approach zero for large values of τ, as they should; and that the control force magnitude $T(\tau)$ and orientation angle $\beta(\tau)$, approach constant values. The quantities T and β are directly related to the control force components F_ξ and F_θ as follows

$$T\cos\beta = -F_\xi \quad T\sin\beta = F_\theta$$

(see Fig. 1, with $\phi \equiv 0$ and $F_\phi \equiv 0$); their limiting values $T = 0.00174$ and $\beta = 0$, respectively, represent the system equilibrium control force $F_\xi = -3m^*\omega^2 l$, $F_\theta = 0$ defined by Eq. (15) and the terminal state vector (12). This is the tension force $T = 3m^*\omega^2 l$ that must be provided by the connecting link, or along the connecting line, of a dual satellite system of end masses m_1 and m_2, and of operational baseline length l, if this system is to move with constant angular velocity $\dot{\theta} = \omega$ in a circular orbit in continuous alignment with the local vertical. For the nominal system values $m_1 = 15.55$ slugs, $m_2 = 9.33$ slugs, $\omega = 7.25 \times 10^{-5}$ rad/sec and $l = 18{,}240$ ft used exclusively in the paper the value of this force is 0.00174 lb.

The results presented in Figs. 2 and 3 are typical of those encountered over a wide range of initial conditions. A number of additional runs were made for different values of the parameters α, κ, and λ of Eq. (17). They confirm the validity of the analysis presented in the paper, in that control force functions of the form of Eq. (15) are guaranteed to drive the (planar restricted) dual satellite system to terminal alignment with the local vertical. They also show (something not illustrated in the paper) that the proportional control factor ν may be utilized to alter the duration of deployment.

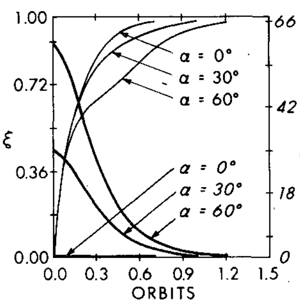

Fig. 2 Deployment paths [Case A; $(\xi_i, \theta_i, \xi_i', \theta_i') = (0, \alpha, 1, 0)$].

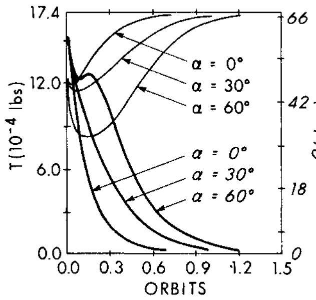

Fig. 3 Control force functions [Case A; $(\xi_i, \theta_i, \xi_i', \theta_i') = (0, \alpha, 1, 0)$].

† An initial separation distance $d \ll l$ between the two satellite mass centers is represented by $\xi_i = 0$ in our mathematical model.

References

[1] Swet, C. J. and Whisnant, J. M., "Deployment of a Tethered Orbiting Interferometer," *Journal of the Astronautical Sciences*, Vol. 27, No. 1, July–Aug. 1969, pp. 44–59.

[2] Bainum, P. M., Harkness, R. E., and Stuiver, W., "Attitude Stability and Damping of a Tethered Orbiting Interferometer Satellite System," *Journal of the Astronautical Sciences*, Vol. 19, No. 5, March–April 1972, pp. 364–389.

[3] Stuiver, W. and Bainum, P. M., "A Study of Planar Deployment Control and Libration Damping of a Tethered Orbiting Interferometer Satellite," *Journal of the Astronautical Sciences*, Vol. 20, No. 6, May–June 1973, pp. 321–346.

[4] Singh, R. B., "Three-Dimensional Motion of a System of Two Cable-Connected Satellites in Orbit," *Astronautica Acta*, Vol. 18, No. 5, Nov. 1973, pp. 301–308.

[5] Beletskii, V. V. and Novikova, E. T., "Three-Dimensional Motion of Two Coupled Bodies in a Circular Orbit," *Izvestiia Akademiia Nauk, Mekhanika Tverdogo Tela*, No. 5, Sept.–Oct. 1971, pp. 23–28; see also *Proceedings of the XXIInd International Astronautical Congress*, Brussels, Oct. 1971.

[6] Meirovitch, L., *Methods of Analytical Dynamics*, McGraw-Hill, New York, 1970, pp. 94–95.